The Effect of Spurious and Missing Minutiae on Delaunay Triangulation Based on Its Application to Fingerprint Authentication

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Abstract—Delaunay triangulation based structures, e.g., the Delaunay triangle and Voronoi neighbor structure, have been utilized in fingerprint authentication due to their desirable features in terms of dealing with non-linear distortion and spurious and missing minutiae. However, no existing research has shed light on the effect of spurious and missing minutiae on Delaunay triangulation when it is applied to fingerprint authentication. In this paper, we provide some straightforward information as to how much effect spurious and missing minutiae can have on Delaunay triangulation. Our investigation is supported by the results of experiments carried out on two common variants of Delaunay triangulation in four different cases. The experimental results show that Delaunay triangulation based structures are more sensitive to missing minutiae than spurious minutiae.

Keywords- Delaunay triangle, Voronoi neighbor structure, fingerprint authentication, spurious and missing points.

I. INTRODUCTION

Biometric authentication, which is based on an individual’s biometric features, e.g., fingerprint, face, finger-vein and iris, tends to take place traditional authentication such as token- and/or password-based authentication [1-3]. Because of the usage convenience and distinctive features that fingerprints can supply compared to other biometrics, fingerprint authentication is widely used in military and civil applications and occupies a wide range of market.

However, one major drawback of fingerprint authentication compared to token- and/or password-based authentication is that the noisy and diverse nature of biometrics, e.g., non-linear distortion, spurious and missing minutiae points, always exist in the process of fingerprint image capturing [4, 5]. In other words, fingerprint features captured from the same individual at different times tend to be different. To counter the issue of fingerprint uncertainty, some alignment-free stable local minutiae structures, e.g., N-nearest local structural, Delaunay triangulation, are considered in many research works[6-13]. As one kind of these local structures, Delaunay triangulation, which has some ideal local and global characteristics, has drawn considerable attention. Firstly, every minutia in the Delaunay triangulation usually keeps the similar neighbor structure, even if there is a certain degree of non-linear distortion. In other words, it has a stable local structure. Secondly, spurious and missing minutiae only affect the local cells that contain these minutiae. Because of these desirable features of Delaunay triangulation, several variants of Delaunay triangulation have been proposed in existing fingerprint authentication algorithms. For example, In [14], Yu et al. presented a radial structure (which is a variant of Delaunay triangulation) based fingerprint matching algorithm. Local matching is carried out by using the radial structures and followed by a global matching when the local matching between template and query fingerprints is unsuccessful. In [15], Khazaei et al. proposed a fingerprint matching algorithm based on the Voronoi diagram. In this algorithm, a unique central cell is found and used for local matching. With the help of local central cell matching, this algorithm is able to filter unmatched fingerprint pairs instantly. Moreover, another process is activated when local central cell matching fails under nonlinear distortion. In [16], Ceguerra et al. introduced an automatic fingerprint verification system based on Voronoi neighbor structures. In this scheme, the Voronoi neighbor structures act as local features and help to find a reference axis consisting of the center minutia of a Voronoi neighbor structure and one of its neighbor minutiae. Then the reference axis is used to create the global features which are finally taken to calculate the similarity between a pair of template and query fingerprint images. In [17], Soleymani et al. combined Delaunay triangulation and Voronoi diagram together to generate a hybrid matching algorithm. Different from the algorithms mentioned in [14-16] which start with local structure comparison, in this algorithm [17], the comparison of global topological polygons generated from the boundaries of Delaunay triangulation is performed first and then the central Voronoi cells of fingerprints are compared to compute the similarity between template and query fingerprint images.

All the above mentioned algorithms take full advantage of Delaunay triangulation based structures to reduce fingerprint uncertainty. However, no existing research has shed light on the effect of spurious and missing minutiae on Delaunay
triangulation. For example, to what extent can local structures be affected when spurious and missing minutiae are present in a fingerprint image? In this paper, we investigate the effect of spurious and missing minutiae on two variants of Delaunay triangulation, namely, the Delaunay triangle (DT) and Voronoi neighbor structure (VNS). Based on experiments conducted in four different cases, we provide some straightforward information as to how much effect spurious and missing minutiae can have on Delaunay triangulation. This research not only offers a good understanding about the effect of spurious and missing minutiae on Delaunay triangulation based structures, but also provides a basis for choosing suitable Delaunay triangulation based structures in a fingerprint authentication system.

The rest of the paper is organized as follows. Two variants of Delaunay triangulation are introduced in Section II. In Section III, statistical data are presented and analyzed based on our experiments. The conclusion is given in Section IV.

II. TWO VARIANTS OF DELAUNAY TRIANGULATION

Two variants of Delaunay triangulation, namely, Delaunay triangle (DT) and Voronoi neighbor structure (VNS), are introduced in this section. We first give a brief introduction about the generation of Delaunay triangulation and readers can refer to [18] for more details. Given a set of minutiae \( M = (m_1, m_2, ..., m_n) \), each minutia \( m_{i,(X,Y)} \) can be represented by a vector \( (x_i, y_i, \theta_i) \), where \( x_i \) and \( y_i \) are positions in the Cartesian coordinate and \( \theta_i \) is the orientation. A Voronoi tessellation divides the whole fingerprint image into many small regions centering at each minutia as shown in Figure 1a. All the minutiae in the region around \( m_i \) are closer to \( m_i \) than to any other minutiae point in other regions. The Delaunay triangulation is generated by connecting the centers of every neighboring regions as shown in Figure 1b.

![Fig. 1. (a) Voronoi tessellation, (b) Delaunay triangulation (bold line)](image)

A. Delaunay Triangle

Each Delaunay triangle (DT) is composed of three neighboring minutiae and any two of these three points share the common edge with each other. For a Delaunay triangulation net generated by the minutia set \( M = (m_1, m_2, ..., m_n) \), \( (2n-N-2-K) \) Delaunay triangles are obtained, where \( K \) is the number of minutiae on the convex hull of Delaunay triangulation. An example of the Delaunay triangle \( \triangle ABC \) is given in Figure 2.

![Fig. 2. An example of the Delaunay triangle \( \triangle ABC \) and Voronoi neighbor structure \( \text{VNS}_A \) centered at point A](image)

B. Voronoi Neighbor Structure

The Voronoi neighbor structure (VNS) used in our implementation is composed of those minutiae that share a common vertex in Delaunay triangulation. For example, the Voronoi neighbor structure \( \text{VNS}_A \) that centers at minutia \( A \) is made up of the center minutia \( A \) and its neighboring minutiae \( B, C, D, E, F, G, H, I \) and \( J \), as shown in Figure 2. Since each minutia in a fingerprint image can be considered as a center of a VNS, \( N \) VNSs can be created, if there are \( N \) minutiae in the fingerprint image.

III. EFFECT OF SPURIOUS AND MISSING MINUTIAE ON TWO VARIANTS OF DELAUNAY TRIANGULATION

In this section, we will show the effect of spurious and missing minutiae on two common variants of Delaunay triangulation, namely, the Delaunay triangle and Voronoi neighbor structure through experiments.
Even though non-linear distortion exists in a fingerprint image, in this paper we do not consider the influence of non-linear distortion on the structure of Delaunay triangulation. In the experiments, we choose the first fingerprint image “1_1.tif” from the public database FVC2002 DB2 as the reference subject. “1_1.tif” has the dimension of “296x560” and a set of 31 minutiae, denoted by $M_{org}$, can be extracted from it by using the software VeriFinger 4.0 from Neurotechnology [19]. To test the effect of spurious and missing minutiae on the Delaunay triangle and Voronoi neighbor structure, four different cases are designed and experimented on each of these two structures. The four cases are designed as follows:

Case 1: 10% of $M_{org}$ (about three) spurious minutiae are added to $M_{org}$ and the final minutiae set is named $M_{S\_10}$.

Case 2: 20% of $M_{org}$ (about six) spurious minutiae are added to $M_{org}$ and the final minutiae set is named $M_{S\_20}$.

Case 3: 10% of $M_{org}$ (about three) genuine minutiae are removed from $M_{org}$ and the final minutiae set is named $M_{M\_10}$.

Case 4: 20% of $M_{org}$ (about six) genuine minutiae are removed from $M_{org}$ and the final minutiae set is named $M_{M\_20}$.

In Cases 1 and 2, three and six randomly generated spurious minutiae are added to the minutiae set $M_{org}$, respectively. And in Cases 3 and 4, three and six genuine minutiae are randomly chosen and removed from the minutiae point set $M_{org}$. The Delaunay triangulation formed by these four resultant minutiae sets ($M_{S\_10}, M_{S\_20}, M_{M\_10}$ and $M_{M\_20}$) are shown in Figure 3 and the points circled by the green circles are the spurious or missing minutiae.

The structures generated by the original minutiae set $M_{org}$ are to be compared with the structures generated from other four minutiae sets ($M_{S\_10}, M_{S\_20}, M_{M\_10}$ and $M_{M\_20}$). We calculate the percentage of the total amount of structures from $M_{org}$ that remain unaltered. Since non-linear distortion is not taken into account in our experiment, for two Delaunay triangles to match, their topologies must be identical. For any two Voronoi neighbor structures to match, one of the following two conditions must be satisfied:

Condition 1: The topologies of two VNSs are the same.

Condition 2: At most only one of the minutiae is different between two VNSs.

The second condition above is reasonable because each VNS has several neighboring minutiae. If only one minutia is different, other minutiae can still provide enough discriminative power. However, if one minutia of a Delaunay triangle changes location, the entire topology of the triangle would be different. The effect of spurious and missing minutiae on these two structures in those four cases is illustrated in Table I, from which straightforward information about the effect of spurious and missing minutiae on Delaunay triangulation can be obtained. We can see that the Delaunay triangle is most affected in Case 4 and only 52.94% of the total structures remain unaltered, while the Voronoi neighbor structure under Case 1 is least affected and all (100%) of the structures stay unchanged if shifting one neighboring minutia can be tolerated. Compared to spurious minutiae, missing minutiae have a more negative effect on the structure of Delaunay triangulation, which means that Delaunay triangulation based structures are more sensitive to missing minutiae than spurious minutiae.

![Delaunay triangulation under four different cases](image-url)
TABLE I. EFFECT OF SUPRIOUS AND MISSING POINTS ON VARIANTS OF DELAUNAY TRIANGULATION

<table>
<thead>
<tr>
<th>Structure Types</th>
<th>Case 1 ($M_{s,10}$)</th>
<th>Case 2 ($M_{s,20}$)</th>
<th>Case 3 ($M_{M,10}$)</th>
<th>Case 4 ($M_{M,20}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DT</td>
<td>96.08%</td>
<td>84.31%</td>
<td>76.47%</td>
<td>52.94%</td>
</tr>
<tr>
<td>VNS under condition 1</td>
<td>87.10%</td>
<td>64.52%</td>
<td>48.39%</td>
<td>19.35%</td>
</tr>
<tr>
<td>VNS under condition 2</td>
<td>100%</td>
<td>93.55%</td>
<td>90.32%</td>
<td>58.06%</td>
</tr>
</tbody>
</table>

The effect is shown in terms of the percentage of the total amount of structures in the template that can remain for matching.

IV. CONCLUSION

In this paper, we provide some straightforward information about the effect of spurious and missing minutiae on two variants of Delaunay triangulation. The experimental results show that the Voronoi neighbor structure is more robust than the Delaunay triangle when spurious and missing minutiae are present in a fingerprint image. Our investigation also provides a basis for choosing appropriate local structures in the design of a fingerprint authentication system. Our experiments show that the Delaunay triangulation based structures are more sensitive to missing minutiae than spurious minutiae, which points to a future research direction. We should research how to counter the negative influence of missing minutiae.

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